**Elicitation of Preferences Under Ambiguity**

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**Abstract**

This paper is about behaviour under *ambiguity* ‒ that is, a situation in which probabilities either do not exist or are not known. Our objective is to find the most empirically valid of the increasingly large number of theories attempting to explain such behaviour. We use experimentally-generated data to compare and contrast the theories. The incentivised experimental task we employed was that of allocation: in a series of problems we gave the subjects an amount of money and asked them to allocate the money over three accounts, the payoffs to them being contingent on a ‘state of the world’ with the occurrence of the states being ambiguous. We reproduced ambiguity in the laboratory using a Bingo Blower. We fitted the most popular and apparently empirically valid preference functionals [Subjective Expected Utility (SEU), MaxMin Expected Utility (MEU) and *α­*-MEU], as well as Mean-Variance (MV) and a heuristic rule, Safety First (SF). We found that SEU fits better than MV and SF and only slightly worse than MEU and *α­*-MEU.

Keywords: allocations, ambiguity, preferences under ambiguity.

JEL codes: C9, D81

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1. *Introduction*

The context of this paper is that of decision-making under ambiguity. Ambiguity is normally considered by decision theorists to be a situation in which in which probabilities either do not exist or are not known. There are now an increasingly large number of theories of behaviour in such situations, and our objective is to look at a subset of these and determine which appears to be most empirically valid. To test between the theories in this subset we use experimentally generated data, asking subjects to allocate money between several accounts, the payoffs to which are ambiguous. This data allows us to fit the various theories and determine which appears to be the ‘best’.

The paper is organised as follows. In Section 2 we summarise the main theories of decision-making under ambiguity, concentrating on those that we think most empirically valid and on which we shall focus. As this paper is about the elicitation of preferences, and because we use a particular elicitation method, we discuss the various alternative elicitation methods in Section 3, and compare their possible properties. In section 4 we state the problem presented to our subjects and possible solutions to it. In Section 5 we describe our experimental implementation. We feel that this implementation is a complement to, and an extension and a refinement of, two apparently closely-related experiments; these we discuss in Section 6, looking at the differences between the various designs. Our results are reported in Section 7 and Section 8 concludes.

1. *Theories of Behaviour Under Ambiguity*

There are many theories of behaviour under ambiguity. A useful survey is that of Etner *et al*. We shall omit a discussion of dynamic models (such as that of Siniscalchi 2009) and hence updating models. We shall also ignore the *Incomplete Preferences* story of Bewley (1986), the *Contraction* model of Gajdos *et al* (2006), the *Variational* model of Maccheroni *et al* (2005), and the *Confidence Function* of Chateauneuf and Faro (2009), partly because of the lack of empirical support and partly because of the difficulty of parameterising these models (these two reasons may well be related).

Historically modelling started simple. If probabilities are not known with certainty, the obvious thing to assume is that there is a range of possible probabilities, with a lower and an upper bound. A pessimist would assume that the worst could happen, and would therefore rank decisions on the basis of their worst-case outcomes – the optimal decision being the one with the least-worst outcome. This is the basis of Wald Maxmin (Bewley 1986). Later it was considered an excessively pessimistic rule and generalised by Arrow and Hurwicz (1972) to *α*-Maxmin, in which decisions are based on a weighted average of the worst and best outcomes. These models worked with raw monetary payoffs.

Then came the revolution of Expected Utility theory in which outcomes are not evaluated on the basis of their monetary value, but on the *utility* of their monetary value. Two models which made the obvious generalisation of Maxmin and *α*-Maxmin are Maxmin Expected Utility (Gilboa and Schmeidler 1989) and *α*-Maxmin Expected Utility (Ghirardato *et al* 2004). In both these theories the decision-maker ‒ the DM ‒ uses the *utility* of the outcomes.

In all the above models ‘worst’ and ‘best’ relate to possible outcomes, and covers a world in which the possible outcomes do not have probabilities attached to them, but can be ranked. But taking away probabilities is too much for most theorists. Indeed, economists who ‘believe’ in Subjective Expected Utility (SEU) can of course continue to assume that.

At the same time, assuming that the DM believes that additive probabilities exist (and uses them) is a strong assumption, particularly in an ambiguous world. A partial softening of that strong assumption (but not interpretable as a total abandonment) is that used in Choquet Expected Utility theory (Schmeidler 1989). In this the DM is thought of as attaching *capacities* to the various outcomes, where crucially these capacities are *non-additive* ‒ so that the capacity attached to the union of two disjoint events, *C(S1S2)*,is not necessarily equal to  *C(S1) + C(S2)*. To avoid violations of dominance these capacities are associated with *ranked* payoffs. This is very similar to the procedure used in *Rank Dependent Expected Utility* theory (Kahneman and Tversky 1992), though here weighted probabilities, rather than capacities, are used.

Some theorists do not like the idea of encoding the ambiguity of an event with a single number (probability or capacity or weighted probability). One route is to say that the probability of some event is not a single number but may be one number from a set of possible probabilities. Clearly this what the *α*-model (and its various antecedents) is assuming, but these just work with the worst and the best from this set. A model which goes further is the *Smooth* Model of Klibanoff *et al* (2005), which says that, if the DM cannot attach a single number to a probability, at least he or she can state the set of possible probabilities, and, moreover, attach probabilities to each member of the set. This is a sort of two-level probability structure, and, if the DM’s preference function is linear in the probabilities, it reduces to (subjective) Expected Utility theory. For this reason Klibanoff *et al* do not assume that the preference function is linear in the probabilities. We note that while this may be theoretically interesting, it is almost impossible to fit empirically – as one needs to estimate all the possible probabilities and the probabilities attached to them.

The Mean-Variance model (MV), beloved by finance theorists, does not fit neatly into the above categorisation. However, if SEU is used, for example combined with a CARA (Constant Absolute Risk Aversion) utility function, and with normally distributed outcomes, we get a decision rule consistent with MV. Unfortunately, in general, MV violates first-order stochastic dominance (Blavattsky 2010), and, as a consequence is not often used by decision theorists. Nevertheless it is a widely used decision rule in finance, and is essentially simple – relying only on a calculation of a mean and a variance of some prospect. Of course to calculate these, the DM needs to know probabilities, or at least, act as if he or she knows the probabilities.

For the various reasons discussed above, we decided to estimate SEU (because of its simplicity, elegance and popularity), MaxMin Expected Utility (MEU) and its generalisation *α­*-MEU (because of their relative simplicity), and Mean-Variance (MV) (because of its popularity in finance). In addition, believing that many of these theories complicate an already complex decision problem, we estimated a simple heuristic rule, Safety First (SF); we describe this later.

1. *Elicitation Methods*

, , the *Bomb-Risk Elicitation Task* (Crosetto and Filippin 2010), andSome of these are contrasted and compared in Loomes and Pogrebna (2014) and in Zhou and Hey (unpublished). We describe them briefly here.

In the *Holt-Laury* *Price List* method, while the detail may vary from application to application, the basic idea is simple: subjects are presented with an ordered list of pairwise choices and have to choose one of each pair. The list is ordered in that one of the two choices is steadily getting better or steadily getting worse as one goes through the list. Because of the ordered nature of the list, subjects *should* choose the option on one side up to a certain point thereafter choosing the option on the other side. Some experimenters force subjects to have a unique switch point; others leave it up to subjects. The point at which they switch reveals their attitude to risk. Some commentators suggest that the switch point is dependent on the construction of the list.

A second method is to give a set of *Pairwise Choices,* but separately (not in a list) and not ordered. Indeed, typically the pairwise choices are presented in a random order. Some argue that this method, whilst being similar to that of Price Lists, avoids some potential biases associated with ordered lists.

A method which is elegant from a theoretical point of the view is the *Becker-DeGroot-Marschak* *Mechanism*. The method centres on eliciting the value to a subject of a lottery – if we know the value that a subject places on a lottery with monetary outcomes, we can deduce the individual’s attitude to risk over money. Let us discuss one of the two variants of this mechanism that are used in the literature ‒ where the subject is told that they do not own the lottery, but have the right to buy it. The subject’s valuation of the lottery as a potential buyer is the *maximum* price for which they would be willing to buy it. The method works as follows: the subject is asked to state a number; then a random device is activated, which produces a random number between the lowest amount in the lottery and the highest amount. If the random number is less than the stated number, then the subject buys the lottery at a price equal to the random number (and then plays out the lottery); if the random number is greater, then nothing happens and the subject stays as he or she was. If the subject’s preference functional is the expected utility functional, then it can be shown that this mechanism is incentive compatible and reveals the subject’s true evaluation of the lottery. The problem is that subjects do seem to have difficulty in understanding this mechanism, and a frequent criticism is that subjects understate their evaluation when acting as potential buyers and overstate it when acting as potential sellers.

In the *Bomb-Risk Elicitation Task* subjects decide how many boxes to collect out of 100, one of which contains a bomb. Earnings increase linearly with the number of boxes accumulated but are zero if the bomb is also collected.  The authors claim that “this task requires minimal numeracy skills, avoids truncation of the data, allows [us] to precisely estimate both risk aversion and risk seeking, and is not affected by the degree of loss aversion or by violations of the Reduction Axiom.”

The *Allocation* method involves giving the subject some experimental money to allocate between various states of the world, with specified probabilities for the various states, and, in some implementations, with given exchange rates between experimental money and real money for each of the states.

As we have noted above, the different methods have their advantages and disadvantages. In evaluating and comparing them there is a fundamental problem: the experimenter does not know the ‘true’ attitude to risk of the subjects, nor their ‘true’ preference functional. All we can conclude from Loomes and Pogrebna (2010) and Zhou and Hey (unpublished) is that context matters. Further work needs to be done to discover how and why. In the meantime, this paper will use the Allocation method, which is relatively under-used, and, in our opinion, relatively easy for subjects to understand. We describe below the particular allocation problem presented to our subjects.

1. *The Allocation Problem and Possible Solutions*

The problems presented to our subjects took the following form: the subject is given an endowment(which we normalise here to 100, as was the case in our experiment) in cash to allocate to three accounts: one with a certain return (which we normalise to 1); and the other two with uncertain returns, which depend upon which *state of nature* occurs. The number of such states is set at 3, which makes the problem a meaningful[[1]](#footnote-1) one while reducing its complexity. Denote by *c1* and *c2* the allocations to the two uncertain accounts 1 and 2 respectively. This implies that the allocation to the certain account *c0* is given by *c0 = 100 – c1 – c2*. Crucial to the allocation problem are the returns in the uncertain states. Denoting by *rij* the *absolute* return on account *i* if state *j* occurs, we have the following *returns table*:

|  |  |  |  |
| --- | --- | --- | --- |
|  | state 1 | state 2 | state 3 |
| account 1 | *r11* | *r12* | *r13* |
| account 2 | *r21* | *r22* | *r23* |

It follows that the payoff to the subject in state *j*, denoted by *dj*,is given by *dj = c0 +r1jc1 + r2jc2  (j=1,2,3).*

The DM’s optimal allocations depend upon his or her preferences. If we start with Expected Utility (EU) theory under risk, or Subjective Expected Utility (SEU) under ambiguity, where *pj (j=1,2,3)* is the (subjective) probability of state *j* occurring, then the DM’s objective function is the maximisation of *p1u(d1) + p2u(d2) + p3u(d3)* where *u(.)* is the individual’s utility function. If instead the DM follows Mean-Variance (MV) theory using probabilities *pj (j=1,2,3),* then the objective is the maximisation of *μ – rσ2*,where *r* indicates the attitude to risk and the mean, *μ,* and variance, *σ2*, of the portfolio are given by *μ = p1d1 + p2d2 + p3d3* and *σ2 = p1(d1-μ)2 + p2(d2-μ)2 + p3(d3-μ)2* .

The above assumes that the subject works with either objective or subjective probabilities. If, however, the DM feels in a situation of ambiguity and hence unable to attach unique probabilities to the various states of the world, then to model his or her behaviour we need to turn to one of the new theories of behaviour under ambiguity. In this paper we work with the simplest – MaxMin Expected Utility (MEU) and the α-MEU model. Both of these theories start by assuming that, while the DM cannot attach unique probabilities to the various states, he or she works with a *set of possible probabilities.* The theories do not say how this set is specified. We assume what appears to be the simplest: this set is all possible probabilities defined by (non-negative) *lower bounds p1, p2* and *p3* (where *p1 + p2 +* *p3 ≤ 1*)on the probabilities*.* If you like, it is a little triangle properly within the Marschak-Machina triangle.

1. *Our Experimental Implementation*

Subjects were presented with a total of 65[[2]](#footnote-3) allocation problems, in each of which they were asked to allocate 100 in experimental cash to two accounts or to keep some of the 100 as cash. In each of these they were shown a returns table. An example is the following:

|  |  |  |  |
| --- | --- | --- | --- |
|  | pink | green | blue |
| account 1 | 1.7 | 0.9 | 0.6 |
| account 2 | 0 | 0.1 | 3.1 |

The colours represent the possible states of the world and relate to the way that ambiguity was implemented in the experiment. In the laboratory there was a [Bingo Blower](http://www.york.ac.uk/economics/research/centres/experimental-economics/research/ongoing-projects/" \l "tab-2) with pink, green and blue balls blowing around in continuous motion. Subjects could see the balls, and get a rough idea of the numbers and relative proportions of each colour, but, when at the end of the experiment, one ball was ejected by them, they could not be sure of the probability of getting a ball of a particular colour. (There were actually 10 pink, 20 green and 10 blue balls in the Blower, so the objective probabilities were 0.25, 0.5 and 0.25.) Subjects were paid on a randomly chosen problem, with their payment being determined by the payoff (given their chosen allocations) for the state implied by the colour of a ball randomly ejected from the Blower.

A screen shot from the experiment can be seen in Figure 1[[3]](#footnote-4); the ‘returns table’ was called the ‘Payoff Table’. The triangle shows the set of all allowable allocations; as the subject moved his or her cursor around the triangle the ‘Portfolio’ entries on the screen dynamically changed, and the implied payoffs for each colour were shown in the entries under ‘Portfolio Payoff’. Subjects were forced to spend a minimum time of 30 seconds before registering their choice on any problem; there was a maximum time of 180 seconds per problem, and if they had not registered their choice by that time, it was taken to be an allocation of zero to the two uncertain accounts. The instructions given to the subjects can be found [here](http://www.york.ac.uk/economics/research/centres/experimental-economics/research/unpublishedpapers1/" \l "tab-335165-6).

In the experiment we did not allow the subjects to make negative allocations (which they might have wanted to do to maximise their objective function). We enforced this rule to avoid the possibility of subjects making losses in the experiment. This meant that what we observe in the data are not optimal allocations, but optimal *constrained* allocations. In order to fit the various models to the data we need to compute (for any given set of parameters) the optimal constrained allocations. While explicit analytical solutions are obtainable for the optimal unconstrained allocations for some of the preference functionals, they are not easily obtained for the optimal constrained ones. As a consequence we calculate them numerically.

There was also an additional ‘constraint’ on the allocations that subjects could make. In the experiment, the endowment in each problem was 100, and subjects were forced to implement allocations to the nearest integer. Given the non-negativity constraint this implied a set of 5151 possible allocations. Searching over these 5151 possible allocations proved to be a more efficient method of finding the optimal constrained allocations than using some built-in function, because of the complexity of the problem.

1. *Similar Experiments*

proceed to our data analysis

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*6. Results*

We have explored a number of different specifications and we report here just the best. Our primary concern is about the best fitting preference functional; we start with that. We measure the goodness-of-fit by the Maximised Log-Likelihood (MLL), but we need to correct for the number of parameters in the preference functional – the number of degrees of freedom in the estimation.

We have already mentioned the preference functionals we have fitted. Each of these involves a utility function; we have taken two utility functionals. The first is the *Constant Absolute Risk Aversion (CARA)* form so that utility *u(x)* is proportional to *-e-rx.* The second is the *Constant Relative Risk Aversion (CRRA)* form so that utility *u(x)* is proportional to x1-r. In order to compare the goodness-of-fit of the different specifications, we need to distinguish between pairs of preference functionals one of which is nested within the other, and pairs of preference functionals where neither is nested within the other. We use the *Likelihood Ratio Test* for the former and the Clarke test for the latter. We note that SEU is nested within both MEU and *α*-MEU and that MEU is nested within *α*-MEU, but that none of the other functionals are nested within any other.

We had a total of 77 subjects. We omit 2 from the analysis that follows as they were extremely risk-averse, investing nothing in either risky account[[4]](#footnote-5). We then divide the remaining 75 subjects into two groups, which we call the CARA-better group and the CRRA-better group, membership of which was determined by the value of the maximised log-likelihood. For 71 of these 75 subjects, one of CARA or CRRA had a higher log-likelihood[[5]](#footnote-6). There are 56 in the CARA-better group and 19 in the CRRA-better. We then report the results of the Likelihood Ratio and the Clarke tests for each of these groups separately.

When one model is nested within another, the test statistic is

where is the maximised log-likelihood of the nested model and is the maximised log-likelihood of the nesting model. The test statistic has a Chi-square distribution with degrees of freedom equal to the difference in the number of parameters in the two competing models. As α-MEU has one more parameter than MEU and as MEU has one more parameter than SEU, the corresponding degrees of freedom for SEU *v* MEU, SEU *v* α-MEU and MEU *v* α-MEU are 1, 2 and 1 respectively. The results are summarised in Table 1, which reports the percentage of the subjects for which the test was significant. Table 1 (A) gives the results for the CARA-better group and Table 1(B) gives the results for the CRRA-better group.

**Table 1: Percentage of subjects significant using the Likelihood Ratio Test**

**(A) CARA-better group**

|  |  |  |
| --- | --- | --- |
|  | significant at 5% | significant at 1% |
| MEU *v* SEU | 18% | 14% |
| α-MEU *v* MEU | 13% | 9% |
| α-MEU *v* SEU | 25% | 13% |

**(B) CRRA-better group**

|  |  |  |
| --- | --- | --- |
|  | significant at 5% | significant at 1% |
| MEU *v* SEU | 11% | 11% |
| α-MEU *v* MEU | 21% | 11% |
| α-MEU *v* SEU | 27% | 11% |

As the results are similar for the two groups, we put them together and note that both MEU and α-MEU do moderately better than SEU for a small number of subjects, which may not be surprising as the decision problem was one under ambiguity rather than under risk. Nevertheless SEU performs well.

When models are *not* nested one within the other we use the *Clarke Test* (Clarke 2007). The null hypothesis is that the models are equally good, and hence on a particular problem the probability of the log-likelihood for one model being larger than the probability of the other model is ½. That is:

Here and are the individual log-likelihoods of the 65 problems, which are calculated using the estimated parameters of the two competing models. The test statistic is

where

Under the null hypothesis *T* has a binomial distribution with parameters *n=65* and *p=0.5.* Thus an observation greater than 40 or less than 25 rejects the null hypothesis at the 5% significance level. The results are summarised in Table 2. These are the *percentages* for which the test was significant. Table 2 (A) gives the results for CARA-better group and Table 2 (B) gives the results for CRRA better-group.

**Table 2: Clarke Tests**

**(A) CARA-better group**

(a) Comparisons between SF, SEU, MEU and α-MEU (5% significance level)

|  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- |
| SEU *v* SF | | | MEU *v* SF | | | α-MEU *v* SF | | |
| SEU better than SF | SF better than SEU | Neither better than the other | MEU better than SF | SF better than MEU | Neither better than the other | α-MEU better than SF | SF better than α-MEU | Neither better than the other |
| 70% | 5% | 25% | 70% | 2% | 28% | 70% | 2% | 28% |

(b) Comparisons between MV, and SEU, MEU and α-MEU (5% significance level)

|  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- |
| SEU *v* MV | | | MEU *v* MV | | | α-MEU *v* MV | | |
| SEU better than MV | MV better than SEU | Neither better than the other | MEU better than MV | MV better than MEU | Neither better than the other | α-MEU better than MV | MV better than α-MEU | Neither better than the other |
| 50% | 7% | 43% | 48% | 4% | 48% | 52% | 5% | 43% |

**(B) CRRA-better group**

(a) Comparisons between SF, SEU, MEU and α-MEU (5% significance level)

|  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- |
| SEU *v* SF | | | MEU *v* SF | | | α-MEU *v* SF | | |
| SEU better than SF | SF better than SEU | Neither better than the other | MEU better than SF | SF better than MEU | Neither better than the other | α-MEU better than SF | SF better than α-MEU | Neither better than the other |
| 63% | 0% | 37% | 63% | 0% | 37% | 63% | 0% | 37% |

(b) Comparisons between MV, and SEU, MEU and α-MEU (5% significance level)

|  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- |
| SEU *v* MV | | | MEU *v* MV | | | α-MEU *v* MV | | |
| SEU better than MV | MV better than SEU | Neither better than the other | MEU better than MV | MV better than MEU | Neither better than the other | α-MEU better than MV | MV better than α-MEU | Neither better than the other |
| 74% | 0% | 26% | 79% | 0% | 21% | 84% | 0% | 16% |

Here there are more noticeable differences between the two groups. In a comparison between SF, SEU, MEU and α-MEU, SF does not perform too well in the CARA-better group, though it does marginally better in the CRRA-better group. In comparisons between MV, SEU, between MEU and MV and between α-MEU and MV, in the CARA-better group SEU is often significantly better than MEU and α-MEU, and very rarely is one of the more general functionals significantly better than SEU. In the CRRA-better group, SEU does even better.

As a side issue, it may be interesting to report on the estimated probabilities for SEU and the estimated lower bounds on the probabilities for MEU and α-MEU; recall that the true probabilities were 0.25 (pink), 0.5 (green) and 0.25 (blue). When the CARA utility functional is the one estimated, the averages (over all subjects) of the estimated probabilities for SEU were 0.262, 0.530 and 0.208, which are very close to the true probabilities (though there was considerable dispersion across subjects). For MEU the average lower bounds were 0.228, 0.507 and 0.190, while for α-MEU they were 0.212, 0.490 and 0.171. These are (necessarily) lower than the corresponding SEU probabilities, but only marginally so. These figures suggest that while, for some subjects, MEU or α-MEU are *statistically* superior to SEU, the economic importance is marginal. When the CARA utility functional is the one estimated, these numbers are 0.257, 0.514 and 0.229 for SEU; 0.233, 0.503 and 0.233 for MEU; 0.224, 0.462 and 0.198 for α-MEU. These are very similar to those when the CARA functional was that estimated.

While SF does not perform particularly well, it may be if interest to report the estimated values of the threshold *w* – the distribution is in Figure 2. It will be seen from this that many subjects had a very high threshold – some approaching 100%. This alternatively could be interpreted as the result of very high risk-aversion, but this will of course by picked up by SEU (or MEU or α-MEU) with a high estimated level of risk-aversion.

1. *Conclusions*































The main conclusion from the experiment is that MV did rather badly as an explanation of behaviour; possibly as a consequence of it being a special case of SEU. In contrast SEU does rather well, not only compared to MV, but also compared with the generalisations, MEU and α-MEU: for relatively few subjects do these latter perform better. This indicates that subjects do not use a more complicated preference functional when choosing their allocations in a complicated setting. At the same time our simple rule, SF, does worse than SEU, suggesting some sophistication in subjects’ decisions. Finally, it is reassuring for experimentalists that the results of Ahn *et al* and Hey and Pace (2014) are confirmed by ours, insofar as they are comparable.

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*Figures*

Figure 1: A screen shot from the experiment

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Figure 2: The distribution of the estimated threshold *w* for SF

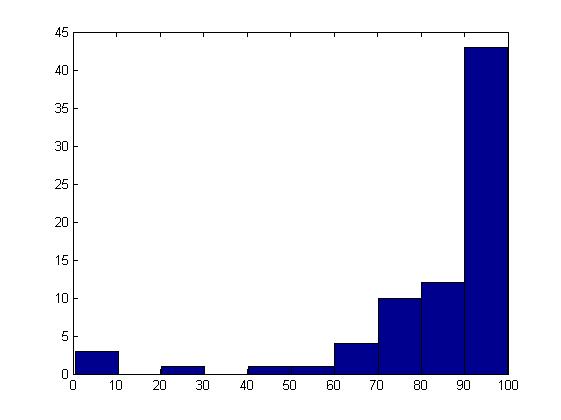


Table 3: Differences between this paper and those of Ahn *et al* (2014) and Hey and Pace (2014)

|  |  |  |  |
| --- | --- | --- | --- |
| Topic | This paper | Ahn *et al* (2014) | Hey and Pace (2014) |
| Econometrics | Maximum Likelihood assuming Beta with bias for two random variables | Non-linear least squares (NLLS) – normality implicit | Maximum Likelihood assuming Beta with bias for one random variable |
| Models | SEU, Mean-variance, MaxMin, α-MEU, SF | Kinked and Smooth – others mentioned in an Appendix | SEU, CEU, AEU\*, VEU, COM\*\*  (\* same as α-MEU \*\*Contraction Model) |
| Utility function | CARA and CRRA | CARA | CRRA |
| Setting | 3 states of the world (colours) – all 3 ambiguous | 3 states of the world – 1 risky, 2 ambiguous | 3 states of the world (colours) – all 3 ambiguous |
| Subjects decisions | Allocate between three accounts | Allocate between three accounts | Allocate either (1) between one account and another account or (2) between one account and the other two accounts. |
| Accounts and returns | 1 certain account – with a return of 1.00 in all 3 states of the world, and 2 ambiguous accounts – both pay off something in each state of the world. Asset prices are 1 | The accounts are 3 Arrow securities – each pays 1.00 in just 1 state of the world. Asset prices are not 1. | 1 certain account – with a return of 1.00 in all 3 states of the world, and 2 ambiguous accounts – both pay off something in each state of the world. Asset prices are 1 |
| Ambiguity implementation | Bingo Blower | Subjects told that the ambiguous states ‘were selected with unknown probabilities that sum to ⅔’ | Bingo Blower |
| Finding optimal allocations | Numerical search over (integer) grid. | Calculated analytically (possible because of the assumptions) | Calculated analytically (possible because of the assumptions) |
| Experimental interface | Visual Studio program in which allocations for given cursor position in a triangle are shown and the implications shown alongside | 3-dimensional [representation](http://eml.berkeley.edu/~kariv/ACGK_I_A1.pdf) with planes inserted for prices | Visual Studio program in which subjects indicate preferred allocations with a slider |
| Number of problems | 65 | 50 | 76 |

1. If there were just 2 states there would not be enough information in the data to allow us to infer preferences. [↑](#footnote-ref-1)
2. These problems (and the number of them) were chosen after intensive pre-experimental simulations based on results from a pilot experiment, and were chosen to maximise the power of our estimates. [↑](#footnote-ref-3)
3. We should note that we worded the instructions so that the decision problem represented an investment problem rather than an allocation problem, as we thought that our subjects would be more familiar with the former. This is also reflected in Figure 1. [↑](#footnote-ref-4)
4. All the models, with appropriate parameters, can equally well describe the behaviour of these 2 subjects. [↑](#footnote-ref-5)
5. The allocation of the final 4 was done on the basis of a majority rule. [↑](#footnote-ref-6)